Tentamen Kansrekening

08-07-2010, (No book or calculator).

1. I throw a die till I obtain a 4.

   a) What is the expected number of throws I have to perform?

   b) What is the expectation of the number of eyes I throw in total (before I throw the 4)? Use Wald's formula.

2. a) Give an example of a distribution function which one can neither describe by using only a probability mass function nor by using only a probability density.

   b) Let on the interval between 0 and π a function be given by $f(x) = C\sin x$. For which value of C is this function a probability density?

   c) Give an example of a pair of random variables X,Y which are uncorrelated but not independent.

3. I have three urns, one with 4 blue balls and 2 green balls, one with 4 green balls and 2 blue balls and one with only blue balls. I choose an urn with probability $\frac{1}{3}$, and draw four times a ball from the urn, while putting them back after each draw.

   a) What is the probability that I draw k green balls, for $k = 0, 1, 2, 3, 4$?

   b) Given that I drew 3 blue balls, what is the probability that they originated from urn 1, cq urn 2, cq urn 3?

4. A radioactive isotope has a half-life of half an hour, that is each atom decays with probability one half within that period. What is the expectation value and what the variance of the survival time of such an atom?

5. At a game of bridge each player gets 13 cards.

   a) What is the probability that one player has four aces?

   b) What is the probability that at least one of the players has exactly one ace?

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